Momentum and Heat Transfer Correlations for a Reacting Gas in Turbulent Pipe Flow

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Experimental values of radial temperature and velocity in the gaseous equilibrium type of system $N_2O_4 \rightleftarrows 2NO_2$ in fully developed turbulent flow in a tubular heat exchanger have been found to follow correlations applicable to nonreacting systems. Though thermal properties such as isobaric specific heat capacity and thermal conductivity are nonmonotonic functions of temperature, Deissler's analogy for constant fluid properties is found to be applicable if a dimensionless specific enthalpy H^+ is used in place of t^+ in plotting vs. y^+. The Sieder-Tate heat transfer correlation with a coefficient of 0.018 instead of 0.023 is applicable if specific enthalpy is used as a driving force in place of temperature driving force in the Nusselt number. The effect of dissociation on nonthermal properties such as density and viscosity is slight so that the ordinary u^+ vs. y^+ correlation and the friction factor-Reynolds number correlation are valid. A range of Reynolds numbers from 11,320 to 19,890 was covered under heating conditions with the tube wall temperatures ranging from 123° to 336°F. at 1 atm.

Heat transfer characteristics of dissociating fluids are of interest in both the chemical process and space technology fields. A review of existing literature for heat transfer measurements in flowing fluids is given by Smith et al. (*5, 6*). To date point values of temperature and velocity with the $N_2O_4 \rightleftarrows 2NO_2$ system have been obtained only in the experimental study of Krieve and Mason (*7*), and it is the purpose of this paper to test these data for conformance to conventional correlations for momentum and heat transfer. In this study radial temperature profiles and static pressure were measured at four stations located 1¾, 10, 29½, and 47¾ in. downstream (Stations 1 through 4) in a 49½ in. length horizontal heated tube 0.563 in. I.D. Radial velocity measurements were made with a pitot tube at Stations 2 and 4, but only velocity data at Station 4 where the velocity distributions were fully developed are included. Temperature data for Station 3 are the only ones included since this was the first station downstream where fully developed temperature profiles occurred and there were axial heat losses to the unheated downstream section of the tube at Station 4. The system is assumed to be in complete chemical equilibrium, and the physical properties employed are those presented by Brokaw (*1*).

VELOCITY AND TEMPERATURE PROFILES

Expressions for universal velocity and temperature profiles are of value both for predicting local values of velocity and temperature in a fluid as well as for calculating overall momentum and heat transfer rates. A comprehensive review of the problem is given by Deissler (*3*). His analysis shows that universal velocity and temperature profiles exist which near the walls are expressed in terms of dimensionless variables, where

$$u^+ = t^+ = y^+ \tag{1}$$

$$u^+ = \frac{u_y}{(\tau_w/\rho_w)^{1/2}} \tag{2}$$

$$t^+ = \frac{(t_w - t_y)(C_p\,\tau)_w}{(q/A)_w(\tau/\rho)_w{}^{1/2}}\,g_c \tag{3}$$

$$y^+ = \frac{(\tau/\rho)_w{}^{1/2}}{(\mu/\rho)_w}\,y \tag{4}$$

At distances ($y^+ \geqslant 26$) away from the wall the profiles are given by

$$u^+ = t^+ = a \ln y^+ + b \tag{5}$$

where a and b are constants dependent on the Prandtl number of the fluid and a parameter β which accounts for variable fluid properties:

$$\beta = \frac{1}{t^+}\left(1 - \frac{t_y}{t_w}\right) \tag{6}$$

In this paper only the case of $\beta = 0$ (constant fluid properties) will be considered. Shear stress at the wall τ_w was calculated from pressure drop measurements (*7*) corrected for acceleration of the fluid by adding the term $\frac{G^2}{g_c}\left(\frac{1}{\rho_3} - \frac{1}{\rho_2}\right)$ to experimental values of $(p_3 - p_2)$.

The theoretical u^+ curves defined by Equations (1) and (5) are plotted in Figure 1 for $\beta = 0$. Included in Figure 1 are experimental u^+ data for the equilibrium type of reacting $N_2O_4 \rightleftarrows 2NO_2$ system. The u^+ data agree well with theory. In Figure 2 corresponding t^+ data are plotted, and it is seen that a great deal of scatter

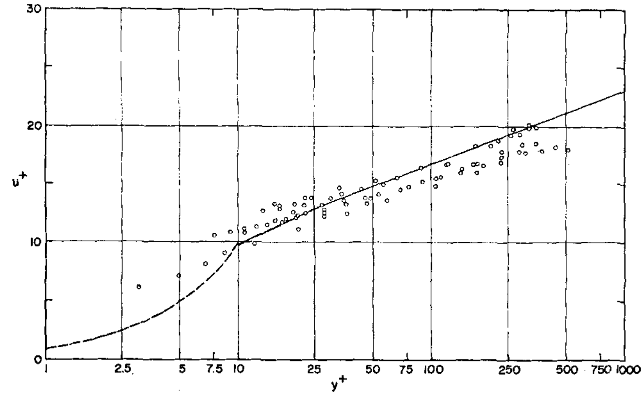

Fig. 1. u^+ **vs.** y^+ **with** $\beta = 0$; $N_{Pr} = 1$. **○ — experimental data substituted in Equations (2) and (4) (reference 9), — — — — — — laminar sublayer [Equation (1)], ———— — turbulent core [Equation (5)].**

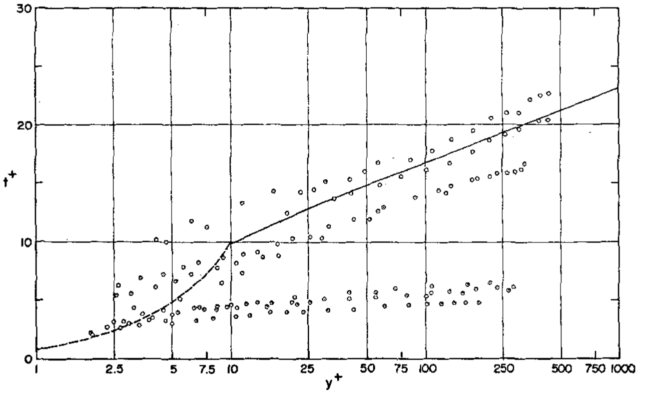

Fig. 2. t^+ **vs.** y^+ **with** $\beta = 0$; $N_{Pr} = 1$. **○ — experimental data substituted in Equations (3) and (4) (reference 9), — — — — — — laminar sublayer [Equation (1)], ———— — turbulent core [Equation (5)].**

occurs, denoting disagreement with theory. For those runs where the points lie below the theoretical curve in Figure 2, the tube wall temperatures were high (263° to 336°F.) corresponding to a high nitrogen dioxide composition and low values of C_{pw} which were substituted in Equation (3). However the average state of the film next to the wall was close to the maximum of k (and C_p) leading to large values of $(q/A)_w$ which were substituted in Equation (3). These composite effects tend to make the experimental values of t^+ lower than the theoretical values. The data are summarized in tabular form in (9).

TURBULENT MOMENTUM—AND HEAT TRANSFER COEFFICIENTS

Expression for the transport of momentum and heat can be obtained from the models leading to the u^+ and t^+ relationships or by the use of the Rayleigh method of dimensional analysis together with experimentation. Several expressions for momentum transfer (4a) in smooth tubes in the turbulent regime are of the form

$$f = -\frac{2D}{U^2\rho}\left(\frac{dP}{dL}\right)_f = 0.184\left(\frac{DG}{\mu}\right)^{-0.2} \qquad (7a)$$

$$f = -\frac{2D}{U^2\rho}\left(\frac{dP}{dL}\right)_f = 0.00560 + \left[\frac{0.5}{\left(\dfrac{DG}{\mu}\right)^{0.32}}\right] \qquad (7b)$$

and for heat transfer (4b) in the turbulent regime

$$N_{Nu} = \frac{hD}{k} = (B)\left(\frac{DG}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14} \qquad (8)$$

where h, the heat transfer coefficient, is defined in terms of the local heat flux density and temperature driving force by

$$(q/A)_w = h\,(T_w - T) \qquad (9)$$

The constant B for ordinary fluids (4b) has been found to be equal to 0.023. In Figures 3 and 4 are presented the theoretical momentum and heat transfer correlations of Equations (7a), (7b), and (8) and (9), respectively. Included are experimental data for $N_2O_4 \rightleftarrows 2NO_2$. Again it is seen that the momentum data correlate fairly well, whereas the heat transfer data in many cases deviate markedly from the conventional curve for nonreacting systems. These data are also presented in Table 1. All properties except μ_w were taken at a state of 1 atm. and at the flow-averaged temperature.

SPECIFIC ENTHALPY DRIVING FORCE IN HEAT TRANSFER EXPRESSIONS

The main problem in treating heat transfer data in equilibrium type of chemically reacting gases is the marked dependence of C_p and k on temperature (2). One way of handling this difficulty is to use an analytical expression to describe the temperature dependence of the property such as Deissler does with μ, ρ, and k (3) permitting u^+ and t^+ vs. y^+ curves to be constructed for various value of β. However because C_p and k are not monotonic func-

TABLE 1. SUMMARY OF EXPERIMENTAL VALUES OF LOCAL FRICTION FACTOR AND HEAT TRANSFER COEFFICIENT*

Run	$\dot{m}$ (lb.$_m$/hr.)	t_w (°F.)	t (°F.)	H_w (B.t.u./lb.$_m$)	H (B.t.u./lb.$_m$)	N_{Re}	f	N_{Pr}	N_{Nu}	N'_{Re}	N'_{Pr}	N'_{Nu}
5	20.95	176	129	227	136	16,590	0.0264	0.775	36.1	16,600	0.774	38.2
7	22.66	123	100	125	83	19,890	0.0296	0.658	44.4	19,890	0.658	39.9
8	22.88	266	141	304	159	17,360	0.0255	0.828	23.0	17,380	0.826	39.8
10	18.87	178	128	229	134	14,980	0.0302	0.772	33.4	14,990	0.771	35.4
11	16.93	176	129	225	136	13,400	0.0301	0.775	31.0	13,410	0.774	32.7
13	18.01	336	158	326	188	12,950	0.0307	0.896	17.1	13,050	0.886	39.0
14	15.26	260	148	301	172	11,320	0.0306	0.857	18.2	11,340	0.854	31.3
17	23.50	146	119	169	117	19,240	0.0297	0.734	46.4	19,240	0.735	48.1
21	24.42	272	146	306	168	18,220	0.0270	0.850	24.2	18,250	0.848	44.0

* Data presented herein are for momentum and heat flux densities between Stations 2 and 3.

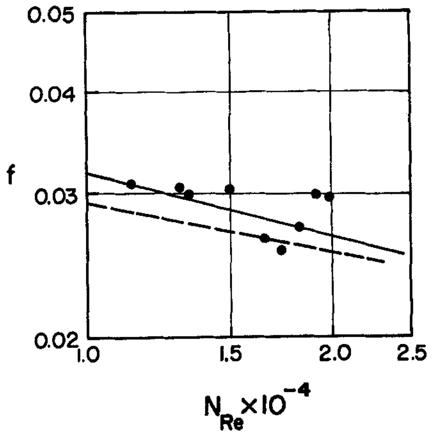

Fig. 3. Momentum transfer correlations. ● — experimental values of f and N_{Re}, - - - - — f vs. N_{Re} [Equation (7a)], ———— — f vs. N_{Re} [Equation (7b)].

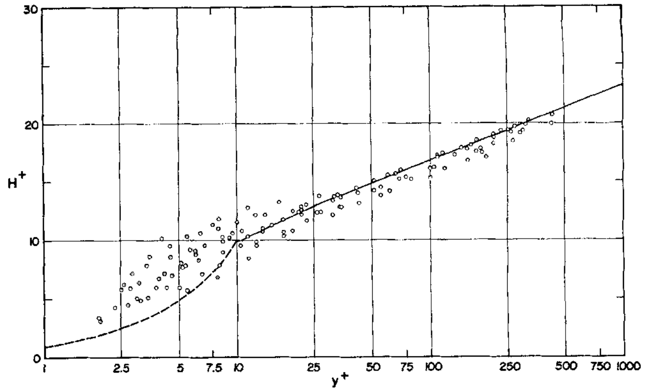

Fig. 5. H^+ vs. y^+ with $\beta = 0$; $N_{Pr} = 1$. ● — experimental data substituted in Equations (10) and (4) (reference 9), - - - - - laminar sublayer [Equation (1) with H^+ replacing t^+], ———— — turbulent core [Equation (5) with H^+ replacing t^+].

tions of temperature (2), no simple expression such as is applicable to nonreacting fluids is possible. Another approach used by Smith et al. (5, 6) is to divide the temperature range into small enough segments that a simple linear expression for the temperature dependence of C_p and k may be employed. However each temperature range requires a different equation. On the other hand if specific enthalpy is used instead of temperature as a driving force, it will be seen that even for $\beta = 0$ (constant fluid properties) reasonably good agreement between experiment and theory results.

The t^+ relationship in Equation (3) may be rewritten in terms of specific enthalpy noting that for a constant pressure (in the radial direction) $dH = C_p\, dT$ and thus

$$H^+ = \frac{(H_w - H_y)\, \tau_w}{(q/A)_w\, (\tau/\rho)_w{}^{1/2}} \tag{10}$$

The assumption of constant C_p inherent in the use of Equation (3) which leads to difficulty is circumvented in Equation (10). In Figure 5 H^+ is plotted vs. y^+, and it is seen the agreement with the theoretical relationship is quite good compared with the approach used in arriving at Figure 2. The driving force Equation (9) may also be rewritten:

$$(q/A)_w = h'\, (H_w - H) \tag{11}$$

Thus the correlation Equation (8) becomes

$$N_{Nu}' = \left(\frac{h'D}{k'/C_p'}\right) = (B)\left(\frac{DG}{\mu}\right)^{0.8} \left(\frac{C_p'\mu}{k'}\right)^{1/3}\left(\frac{\mu'}{\mu_w}\right)^{0.14} \tag{12}$$

It is convenient in applying either Equation (8) or (12)

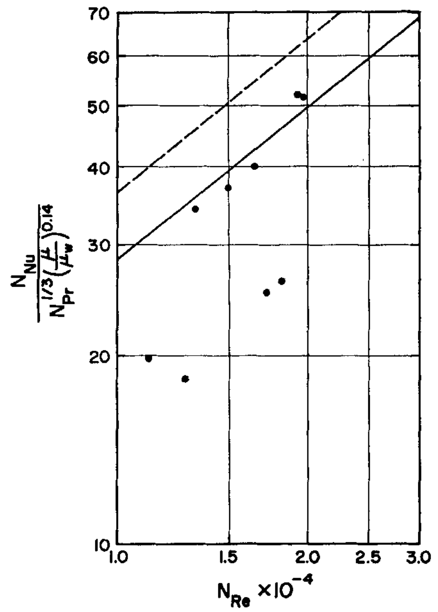

Fig. 4. Heat transfer correlation employing temperature driving force. ● — experimental values of $N_{Nu}/N_{Pr}{}^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$ and N_{Re}, - - - - - Equations (8) and (9) with $B = 0.023$, ———— — Equations (8) and (9) with $B = 0.018$.

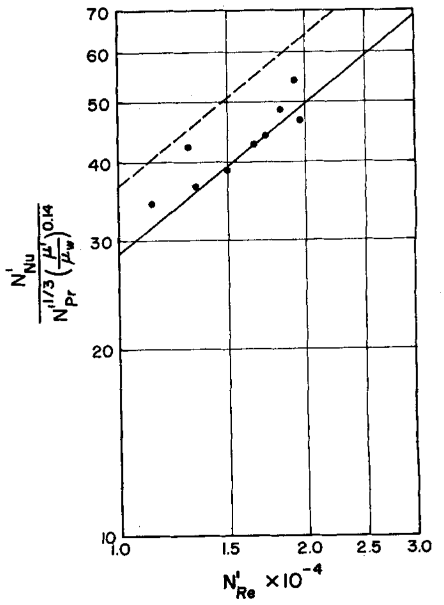

Fig. 6. Heat transfer correlation employing specific enthalpy driving force. ● — experimental values of $N_{Nu}/N_{Pr}{}^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}$ and N_{Re}, - - - - — Equations (11) and (12) with $B = 0.023$, ———— — Equations (11) and (12) with $B = 0.018$.

to employ bulk physical properties since they are most readily known in design problems where the bulk energy is mainly of interest. In Figure 6 are presented the heat transfer correlation of Equation (12) with the constant B assigned to conventional value (4b) of 0.023 and also the value most closely in agreement with the present data, 0.018. All the properties with the exception of μ_w are evaluated at a temperature corresponding to that of the bulk enthalpy. A comparison of Figures 4 and 6 indicates that a better correlation occurs when properties are based on flow averaged specific enthalpy and an enthalpy driving fore is used.

In reacting systems the ratio C_p/k which appears on both sides of Equation (12) is not highly sensitive to temperature as has been previously discussed by Richardson et al. (8). The nonthermal property μ is also not influenced greatly by chemical reaction and thus is not markedly a function of temperature. Thus Equation (12) is fairly insensitive to temperature and could be expected to correlate heat transfer data for reacting systems employing flow-averaged temperature or specific enthalpy. On the other hand the difficulty in the application of Equation (8) is that k, which is so sensitive to temperature, appears singly in N_{Nu} and the assignment of the appropriate value of k between wall and bulk states becomes of critical importance.

CONCLUSIONS

It is seen that the Deissler analogy and conventional momentum and heat transfer correlations hold for this reacting system over a wide range of fluid states; thus the implication is that the Reynolds analogy is applicable (3) or that the eddy viscosity and eddy conductivity are equal in these equilibrium type of systems. A more precise test of the Reynolds analogy however is warranted, and currently precise measurements in a two-dimensional channel with asymmetric wall temperatures and fully developed velocity and temperature profiles are being made; however for engineering accuracy it is seen that these analogies work reasonably well.

To refine the solution the state dependence of nonthermal properties should be taken into account by such a technique as use of Deissler's parameter β. To make these corrections in reacting systems there would be some virtue in using the ratio of the specific enthalpy at a point to that at the wall raised to an empirically established power instead of a temperature ratio. The reason for this approach is that the degree of dissociation (which affects fluid properties due to molecular weight changes) and the specific enthalpy of reacting systems bear a near-linear relationship to one another. This problem is also being worked on.

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NOTATION*

a = constant in Equation (5)
A = circumferential area of tube, sq. ft.
b = constant in Equation (5)
B = constant in Equations (8) and (12)
C_p = isobaric specific heat capacity, B.t.u./lb.-$_m$ °F.
D = internal diameter of tube, ft.
g_c = Newton's law conversion factor, lb.$_m$ ft/lb.$_f$ hr. sq.
G = mass velocity, lb.$_m$/hr. sq. ft.
H = specific enthalpy, B.t.u./lb.$_m$
h = heat transfer coefficient defined in Equation (9), B.t.u./hr. sq. ft. °F.
h' = heat transfer coefficient defined by Equation (11), lb.$_m$/hr. sq. ft.
k = thermal conductivity, B.t.u./hr. sq. ft. °F.
L = axial distance, ft.
$\dot{m}$ = mass flow rate, lb.$_m$/hr.
p = pressure, lb.$_f$/sq. ft.
q = heat flux, B.t.u./hr.
t = temperature, °F.
u = velocity, ft./hr.
U = cross-section average velocity, ft./hr.
y = distance from tube wall, ft.
β = variable fluid property parameter defined by Equation (6)
μ = fluid viscosity, lb.$_m$/ft. hr.
ρ = fluid density, lb.$_m$/cu. ft.
τ = shear stress due to radial momentum transport, lb.$_f$/sq. ft.

Dimensionless Groups

f = friction factor, $-\dfrac{2D}{U^2\rho}\left(\dfrac{dp}{dL}\right)_f$
H^+ = specific enthalpy parameter defined by Equation (10)
N_{Nu} = Nusselt number, hD/k
N'_{Nu} = modified Nusselt number, $h'D/k'/C_p'$
N_{Pr} = Prandtl number, $C_p\mu/k$
N_{Re} = Reynolds number, DG/μ
t^+ = temperature parameter defined by Equation (3)
u^+ = velocity parameter defined by Equation (2)
y^+ = parameter for distance-from-tube wall defined by Equation (4)

Subscripts and Subnumerals

f = axial pressure gradient arising due to friction and excluding acceleration effects, used with dp/dL
w = wall conditions
y = point property at distance y from wall. 2, 3 Axial stations 2 and 3 respectively.

* All symbols for quantities containing properties without subscripts represent the state 1 atm. and either flow-averaged temperature (without primes) or flow-averaged specific enthalpy (with primes).

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